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CONFIDENCE INTERVALS ON NONNEGATIVE LINEAR COMBINATIONS OF VARI--ETC(U)
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by

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ABSTRACT

Smith (1936) suggested a method that can be used for setting confidence limits on linear combinations of variances. This method was studied and expanded by Satterthwaite (1941, 1946) and has become known as Satterthwaite's procedure. The procedure has been widely used for the past 40 years. In this paper a new procedure is proposed for this problem that is better than Satterthwaite's procedure and very easy to compute from existing tables.

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1. INTRODUCTION

Let $n_i S_i^2/\theta_i$ for $i = 1, 2, \ldots, K$ be independently distributed as chi-square random variables with n_i degrees of freedom respectively. A problem which occurs frequently in statistical applications is that of placing confidence intervals on linear combinations of θ_i. For example, consider the one-factor nested components-of-variance model with equal numbers in the subclasses given by $Y_{ij} = \mu + A_i + e_{ij}$ where A_i, e_{ij} are jointly independent, the A_i are distributed $N(0, \sigma_A^2)$ and the e_{ij} are distributed $N(0, \sigma_e^2)$ for $i = 1, 2, \ldots, I$; $j = 1, 2, \ldots, J$. A confidence interval on $\sigma_A^2 + \sigma_e^2$, the total variation, is frequently desired. If S_1^2 and S_2^2 denote respectively the Among and Within mean squares with degrees of freedom n_1 and n_2, and if the expected mean squares are denoted by θ_1 and θ_2, then

$$\sigma_A^2 + \sigma_e^2 = \frac{1}{J}\theta_1 + \frac{J-1}{J}\theta_2$$

is of the form of a <u>positive</u> linear combination of θ_1 and θ_2. Also <u>any</u> nonnegative linear combination of the variance components σ_A^2 and σ_e^2 given by $a\sigma_A^2 + b\sigma_e^2$ may be desired where a and b are given constants such that $a \geq 0$ and $b \geq 0$. Since $c\theta_1 + \theta_2 = c(\sigma_e^2 + J\sigma_A^2) + \sigma_e^2 = cJ(\sigma_A^2 + (c+1)\sigma_e^2/cJ)$, a confidence interval on $c\theta_1 + \theta_2$ with $c \geq 0$ is equivalent to a confidence interval on $a\sigma_A^2 + b\sigma_e^2$ for any specified $a \geq 0$, $b \geq 0$ that satisfies $a/J < b < \infty$.

For another example consider a K - factor nested components-of-variance model with equal numbers in all subclasses (balanced, complete model). The total variation is a positive linear combination of the expected mean squares. Details when K = 3 and K = 4 can be found in several textbooks (e.g., Graybill 1976, Chapter 15).

For a final example consider a two-way crossed classification components-of-variance model with one number in each subclass given by $Y_{ij} = \mu + A_i + B_j + E_{ij}$. Let S_A, S_B, S_E represent the A, B, and E mean squares and θ_1, θ_2, θ_3 represent the corresponding expected mean squares. A quantity that is sometimes needed in this problem is the total variation, $\sigma_A^2 + \sigma_B^2 + \sigma_E^2$. The quantity $\sigma_A^2 + \sigma_B^2 + \sigma_E^2$ is a positive linear combination of the θ_i. Many other problems could be stated in which a positive linear combination of θ_i is required.

There are no exact (the word "exact" means exact specified confidence coefficient) confidence intervals available for nonnegative linear combinations of the θ_i. Smith (1936) defined an estimate of a linear function of variances to be a linear function of independent mean squares and proposed approximating the distribution of such an estimate by a chi-square distribution whose degrees of freedom are determined by equating the variance of the estimate to the variance of the approximating (chi-square) random variable. From this distribution one can obtain approximate confidence intervals on linear functions of variances. Satterthwaite (1941, 1946) studied this approximation and it has become known as the Satterthwaite procedure. Welch (1956) exhibited a series approximation, analogous to the Cornish-Fisher expansion, for the general problem of finding confidence limits for linear combinations of several variances. Huitson (1955) also gave a method for setting confidence intervals on linear combinations of variances. He arrives at some of the methods presented by Welch, although the details of their derivations differ considerably. Huitson includes a special set of tables which must be used to obtain the confidence intervals. Fleiss (1971) discusses the Satterthwaite and Welch methods for setting confidence limits on $\sigma_A^2 + \sigma_e^2$ for the two factor cross component-of-variance model and arrives at the conclusion that Welch's method is adequate (and better than Satterthwaite's method). Fleiss only evaluates the cases where $n_2 = 2n_1$. However, when n_2 is <u>large</u> relative to n_1, Welch's procedure may <u>not</u> be very good. This is demonstrated in Table 1. This table was obtained by numerical integration and the entries are the ranges over which the confidence coefficients vary as the unknown parameter $\rho = c\theta_1/(c\theta_1 + \theta_2)$ varies from 0 to 1. The nominal confidence coefficient is $1 - \alpha = .95$. In component-of-variance models in applied problems it is often the case that n_2 is much larger than n_1 so the conclusions given by Fleiss may not apply in those cases.

Burdick and Sielken (1978) propose a method for constructing exact size confidence intervals for this problem, but the expected lengths of their intervals are extremely bad. They are sometimes more than 800% larger than the expected widths given by Satterthwaite, so their method cannot be recommended for the problem discussed in this paper.

The purpose of this paper is to propose and evaluate a method, called the Modified Large Sample (MLS) method, for obtaining confidence intervals on $\theta = \sum_{i=1}^{K} c_i\theta_i$ with nonnegative constants c_i. The procedure proposed here is compared to those of Satterthwaite and Welch.

2. THE PROPOSED METHOD

In this section we derive a method for setting confidence intervals on $\theta = \sum_{i=1}^{K} c_i\theta_i$. To illustrate the method we first discuss it for a linear combination of two variances, i.e. for $\theta = c\theta_1 + \theta_2$. The UMVU estimator $\hat{\theta}$ of θ is $cS_1^2 + S_2^2$, and $\mathrm{var}[\hat{\theta}] = c^2(2\theta_1^2/n_1) + 2\theta_2^2/n_2$. Thus $Z = (\hat{\theta}-\theta)/\sqrt{\mathrm{var}[\hat{\theta}]}$ has a limiting normal distribution with mean zero and variance one as $\min(n_1,n_2) \to \infty$. Using these results an approximate $1 - \alpha$ confidence interval on θ is given by

$$cS_1^2 + S_2^2 - N_\alpha\sqrt{c^2(2\theta_1^2/n_1) + 2\theta_2^2/n_2} \le \theta \le cS_1^2 + S_2^2 + N_\alpha\sqrt{c^2(2\theta_1^2/n_1) + 2\theta_2^2/n_2}$$

where N_α is the upper α probability point of a standard normal p.d.f.

To utilize these limits, we replace θ_1^2 and θ_2^2 by S_1^4 and S_2^4 respectively. We then modify the confidence limits so they might be more exact for small or moderate sample sizes by replacing the constants $-N_\alpha$, N_α, $2/n_1$, $2/n_2$ by general constants and obtain the following for the approximate $1 - \alpha$ confidence interval on θ

$$cS_1^2 + S_2^2 - \sqrt{L_1^2c^2S_1^4 + L_2^2S_2^4} \le \theta \le cS_1^2 + S_2^2 + \sqrt{H_1^2c^2S_1^4 + H_2^2S_2^4}. \quad (2.1)$$

We now determine L_1, L_2, H_1, H_2 by forcing the confidence interval to have an exact confidence coefficient $1 - \alpha$ when $\theta_1 = 0$ and when $\theta_2 = 0$. When $\theta_i = 0$ it follows that $S_i^2 = 0$ with probability one so we obtain $L_i = 1 - 1/F_{\alpha_{i1}:\, n_i,\, \infty}$, $H_i = 1/F_{\alpha_{i2}:\, n_i,\, \infty} - 1$ for $i = 1, 2$ where $F_{\gamma:\, m,\, n}$ is the upper γ probability point of Snedecor's F distribution with m degrees of freedom in the numerator and n degrees of freedom in the denominator. Also $\alpha_{i1} > 0$, $\alpha_{i2} > 0$, $\alpha_{i1} + \alpha_{i2} = 1$. The resulting confidence interval on $c\theta_1 + \theta_2$, called the Modified Large Sample (MLS) confidence interval, is in (2.1).

The α_{ij} can be chosen so that when $\theta_i = 0$ for either $i = 1$ or $i = 2$ the resulting confidence interval satisfies one of the following three conditions. (1) "Equal tails" confidence intervals (we denote this method by MLS1 and in this case $\alpha_{i1} = \alpha/2$ and $\alpha_{i2} = 1 - \alpha/2$ for $i = 1, 2$); (2) "Shortest unbiased" confidence intervals (we denote this method by MLS2 and for values of L_i, H_i see John (1973)); (3) "Shortest" confidence intervals (we denote this method by MLS3 and for values L_i, H_i see Tate and Klett (1959)).

Note that the confidence interval in (2.1) is also exact when $n_1 \to \infty$ and n_2 is fixed, or when $n_2 \to \infty$ and n_1 is fixed.

To generalize the MLS procedure to nonnegative linear combinations of K variances, we proceed as follows:

a) $U_i = n_i S_i^2/\theta_i$ are independent chi-square random variables with n_i degrees of freedom for $i = 1, 2, \ldots, K$.

b) define θ by $\theta = \sum_{i=1}^{K} c_i \theta_i$ where $c_i \geq 0$, $c_K = 1$;

c) an approximate $1 - \alpha$ confidence interval on θ is

$$\Sigma c_i S_i^2 - \sqrt{\Sigma L_i^2 c_i^2 S_i^4} \leq \theta \leq \Sigma c_i S_i^2 + \sqrt{\Sigma H_i^2 c_i^2 S_i^4} \qquad (2.2)$$

where $L_i = 1 - 1/F_{\alpha_{i1}:\, n_i,\, \infty}$; $H_i = 1/F_{\alpha_{i2}:\, n_i,\, \infty} - 1$ where $\alpha_{i1} > 0$, $\alpha_{i2} > 0$, $\alpha_{i1} + \alpha_{i2} = 1$ for $i = 1, 2, \ldots, K$. The α_{ij} can be chosen for equal tails, for shortest, or for shortest unbiased confidence intervals when $K - 1$ of the θ_i are zero. The confidence interval in (2.2) is exact when (1) any $K - 1$ of the θ_i are zero; (2) when any $K - 1$ of the $n_i \to \infty$. When any M of the $\theta_i = 0$ for $M < K$ the resulting confidence interval reduces to the MLS confidence interval for a nonnegative linear combination of the remaining $K - M$ variances θ_i; also when any M of the $n_i \to \infty$ for $M < K$ the resulting confidence interval reduces to the MLS confidence interval for a nonnegative linear combination of the remaining K - M variances θ_i.

3. THE SATTERTHWAITE, WELCH, AND MODIFIED LARGE SAMPLE METHODS

Let $Z = (cS_1^2 + S_2^2)/(c\theta_1 + \theta_2) = \rho U_1/n_1 + (1-\rho)U_2/n_2$ where $c \geq 0$, $\rho = c\theta_1/(c\theta_1 + \theta_2)$, and U_1 and U_2 are independently distributed as chi-square random variables with n_1 and n_2 degrees of freedom respectively. To determine the distribution of Z we use the following theorem (see Fleiss (1971)).

Theorem 1. The distribution of Z conditional on $W = \frac{U_2}{U_1 + U_2} = w$ is that of $k(U_1 + U_2)$, where $k = \rho(1-w)/n_1 + (1-\rho)w/n_2$ and U_1, U_2 are independent chi-square random variables with n_1 and n_2 degrees of freedom respectively.

The following corollary can be used to evaluate the probability coverages of confidence intervals on $c\theta_1 + \theta_2$ for the Satterthwaite, Welch, and MLS methods.

Corollary 1. Let $z(W)$ be a function of W. Then $P[Z \leq z(W)]$ is

$$P[Z \leq z(W)] = \int_0^1 H_{n_1 + n_2}(z(w)/[\rho(1-w)/n_1 + (1-\rho)w/n_2])p(w)dw,$$

where $p(w) = \frac{1}{B(n_1/2,\ n_2/2)} w^{n_2/2-1}(1-w)^{n_1/2-1}$ is the p.d.f. of W and $H_n(.)$ is the c.d.f. of a chi-square random variable with n degrees of freedom.

The confidence limits of the Satterthwaite, Welch and MLS methods are functions of W so this corollary and numerical integration can be used to evaluate the confidence coefficients as a function of the unknown parameter ρ and specified values of n_1 and n_2.

4. EVALUATION OF CONFIDENCE COEFFICIENTS AND EXPECTED WIDTHS

It is seen that the probability coverages associated with the Satterthwaite, Welch, and MLS approximate confidence intervals for $c\theta_1 + \theta_2$ depend on c and the population parameters θ_1, θ_2 only through the unknown parameter ρ defined by $\rho = c\theta_1/(c\theta_1 + \theta_2)$. Clearly $0 \leq \rho \leq 1$. Simpson's rule with interval size h = 0.01 was used to evaluate the integral (the probability) in Corollary 1 for the different functions z(w) given by the Satterthwaite, Welch, and MLS methods. The IMSL subroutine MDCH was used to compute the chi-square distribution. The values of ρ used were ρ = 0.0 (0.1) 1.0; all combinations of the following values of n_1 and n_2 were examined for $1 - \alpha$ equal to .90 and .95.

n_1: 4, 5, 6, 7, 8, 9, 10, 15, 20, 30

n_2: 4, 5, 6, 7, 8, 9, 10, 15, 20, 30

Tables 2 and 3 contain some of the results. The entries are the ranges that the confidence coefficients vary as the unknown parameter ρ varies in the set 0.0 (.1) 1.0. The column headed MLS1 is for "equal tails" confidence intervals; the column headed MLS2 is for "shortest unbiased" confidence intervals; the column headed MLS3 is for "shortest" confidence intervals. These are defined in Section 2.

To evaluate the expected lengths a simulation study was conducted. One thousand chi-square random numbers were generated using the IMSL subroutine GGCSS (chi-square random deviate generator) for each pair of values of n_1 and n_2 listed below

n_1	4, 4, 4, 8, 8, 16
n_2	4, 8, 30, 8, 30, 32

From these random numbers the three ratios, r_1, r_2 and r_3 were evaluated for $1 - \alpha = .90$ and $1 - \alpha = .95$ where

$$\frac{\text{Average length of MLSi confidence interval}}{\text{Average length of Welch confidence interval}} = r_i$$

The results are recorded in Tables 4 and 5 broken down for $\rho = 0.0(.2)1.0$. The ratios depend on c, θ_1 and θ_2 only through the parameter ρ.

Some conclusions from the formulas are as follows:

(1) The results are for all values of $c \geq 0$.

(2) Only the MLS methods give correct asymptotic results for large n_i and small n_j for $i \neq j$.

(3) When $\rho = 0$ only the Satterthwaite and MLS methods are exact.

(4) When $\rho = 1$ only the Satterthwaite and MLS methods are exact.

(5) The MLS methods are easy to compute even for nonnegative linear combinations of K variances.

Some conclusions from the tables are as follows:

(1) The confidence coefficients for the Welch method are closer to the nominal values than the Satterthwaite method but the Welch method is more difficult to compute.

(2) The confidence coefficients for the Welch and Satterthwaite methods can fall several points below the nominal level. This is undesirable.

(3) The confidence coefficients for the MLS methods appear to be greater than or equal to the nominal values.

(4) The MLS2 and MLS3 methods give confidence intervals whose average widths are generally smaller (and sometimes significantly smaller) than the average widths of the Welch method.

A very small study was conducted for K = 3. An extension of Corollary 1 was used to evaluate the confidence coefficients for the MLS2 and Welch methods. The results are in Table 6 where the entries are the ranges of the confidence coefficients as the unknown parameters vary in the interval [0, 1]. The conclusion is that the MLS2 method is better than the Welch method when confidence coefficients are compared.

From these conclusions it seems that the MLS2 and MLS3 methods are to be preferred over the Welch or Satterthwaite methods for computing confidence intervals on nonnegative linear combination of variances.

Table 1

Ranges of Confidence Coefficients for Welch Method

$1 - \alpha = .95$

n_1	n_2	Range of Confidence Coefficients
4	100	.905 - .956
8	100	.937 - .952

Table 2

Ranges of Confidence Coefficients (Times 10^3)
for Satterthwaite, Welch and MLS Procedures

$1 - \alpha = 0.90$

n_1	n_2	S	W	MLS1	MLS2	MLS3
4	4	886-924	889-907	900-921	900-929	900-946
	6	874-921	886-907	900-921	900-925	900-942
	8	868-919	884-907	900-920	900-924	900-939
	10	865-918	882-909	900-920	900-923	900-937
	30	840-913	869-919	900-919	900-919	900-929
5	5	886-921	893-907	900-917	900-925	900-941
	6	882-920	891-907	900-917	900-923	900-939
	8	877-918	889-906	900-917	900-921	900-936
	10	875-917	889-906	900-917	900-920	900-934
	30	855-912	879-915	900-916	900-917	900-926
6	6	887-919	894-907	900-915	900-921	900-937
	8	883-917	891-906	900-915	900-920	900-934
	10	881-916	892-906	900-915	900-919	900-932
	30	864-911	885-910	900-914	900-915	900-924
8	8	890-915	895-905	900-912	900-917	900-931
	10	888-914	894-905	900-912	900-916	900-928
	30	875-910	891-905	900-912	900-913	900-918
10	10	892-913	896-904	900-910	900-914	900-926
	30	882-908	894-903	900-908	900-911	900-918
30	30	897-905	899-901	900-904	900-905	900-910

Table 3

Ranges of Confidence Coefficients (Times 10^3)
for Satterthwaite, Welch and MLS Procedures

$1 - \alpha = 0.95$

n_1	n_2	S	W	MLS1	MLS2	MLS3
4	4	939-965	938-951	949-961	950-969	950-980
	6	928-963	936-952	950-961	950-967	950-978
	8	921-962	932-953	950-962	950-965	950-976
	10	918-962	932-953	950-962	950-964	950-975
	30	893-958	919-954	950-962	950-963	950-970
5	5	939-963	942-952	949-960	950-966	950-977
	6	935-963	941-953	949-960	950-965	950-976
	8	930-961	938-953	950-960	950-964	950-975
	10	927-961	938-953	950-960	950-963	950-974
	30	907-958	928-964	950-960	950-958	950-968
6	6	939-962	944-953	949-959	950-963	950-975
	8	935-961	942-953	947-959	950-963	950-973
	10	932-960	941-953	950-958	950-962	950-972
	30	916-957	934-961	950-959	950-958	950-967
8	8	941-960	945-953	950-957	950-960	950-971
	10	939-959	944-953	950-957	950-960	950-970
	30	927-956	941-952	950-957	950-958	950-965
10	10	943-958	946-953	950-956	950-959	950-969
	30	934-956	944-952	950-955	950-957	950-964
30	30	948-953	949-951	950-952	950-953	950-958

Table 4

Ratios of Expected Length of MSL to Welch Confidence Intervals on $c\theta_1 + \theta_2$

$1 - \alpha = 0.90$

ρ	$n_1=4,\ n_2=4$			$n_1=4,\ n_2=8$			$n_1=4,\ n_2=30$		
	MLS1	MLS2	MLS3	MLS1	MLS2	MLS3	MLS1	MLS2	MLS3
0.0	1.08	.86	.74	1.01	.91	.83	1.00	.97	.94
0.2	1.03	.83	.71	1.17	1.03	.92	1.20	1.03	.94
0.4	1.18	.94	.81	1.17	.98	.86	.85	.69	.60
0.6	1.19	.95	.81	.95	.77	.67	.76	.61	.53
0.8	1.04	.83	.71	.90	.73	.62	.88	.70	.60
1.0	1.08	.86	.74	1.08	.86	.74	1.08	.86	.74

ρ	$n_1=8,\ n_2=8$			$n_1=8,\ n_2=30$			$n_1=16,\ n_2=32$		
	MLS1	MLS2	MLS3	MLS1	MLS2	MLS3	MLS1	MLS2	MLS3
0.0	1.01	.91	.83	1.00	.97	.95	1.00	.98	.95
0.2	1.03	.92	.84	1.12	1.06	1.01	1.04	1.01	.98
0.4	1.15	1.04	.94	1.07	.98	.91	1.08	1.04	1.00
0.6	1.16	1.04	.95	.98	.89	.81	1.03	.98	.93
0.8	1.03	.93	.84	.98	.88	.80	1.00	.95	.90
1.0	1.01	.91	.83	1.01	.91	.83	1.00	.95	.90

Table 5

Ratios of Expected Length of MLS to Welch Confidence Intervals on $c\theta_1 + \theta_2$

$1 - \alpha = 0.95$

ρ	$n_1=4,\ n_2=4$			$n_1=4,\ n_2=8$			$n_1=4,\ n_2=30$		
	MLS1	MLS2	MLS3	MLS1	MLS2	MLS3	MLS1	MLS2	MLS3
0.0	1.24	.98	.86	1.02	.92	.84	1.00	.98	.95
0.2	1.00	.80	.69	1.20	1.04	.94	1.09	.92	.83
0.4	1.11	.88	.77	1.04	.86	.76	.48	.39	.34
0.6	1.12	.89	.78	.72	.58	.51	.45	.36	.32
0.8	1.01	.80	.70	.78	.62	.54	.76	.60	.53
1.0	1.24	.98	.86	1.24	.98	.86	1.24	.98	.86

ρ	$n_1=8,\ n_2=8$			$n_1=8,\ n_2=30$			$n_1=16,\ n_2=32$		
	MLS1	MLS2	MLS3	MLS1	MLS2	MLS3	MLS1	MLS2	MLS3
0.0	1.02	.92	.84	1.00	.98	.95	1.00	.98	.95
0.2	1.02	.92	.84	1.15	1.09	1.03	1.05	1.02	.98
0.4	1.18	1.06	.97	1.06	.97	.90	1.10	1.06	1.01
0.6	1.19	1.07	.97	.94	.85	.78	1.03	.98	.93
0.8	1.03	.92	.84	.96	.87	.79	.99	.94	.90
1.0	1.02	.92	.84	1.02	.92	.84	1.00	.95	.91

Table 6

Ranges of Confidence Coefficients for Confidence Intervals on $c_1\theta_1 + c_2\theta_2 + \theta_3$

$1 - \alpha = 0.95$

n_1	n_2	n_3	MLS2	Welch
4	4	4	.950-.970	.930-.952
4	4	8	.950-.969	.930-.954
4	8	8	.950-.966	.930-.954
8	8	8	.950-.961	.944-.953

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ABSTRACT

Smith (1936) suggested a method that can be used for setting confidence limits on linear combinations of variances. This method was studied and expanded by Satterthwaite (1941, 1946) and has become known as Satterthwaite's procedure. The procedure has been widely used for the past 40 years. In this paper a new procedure is proposed for this problem that is better than Satterthwaite's procedure and very easy to compute from existing tables.